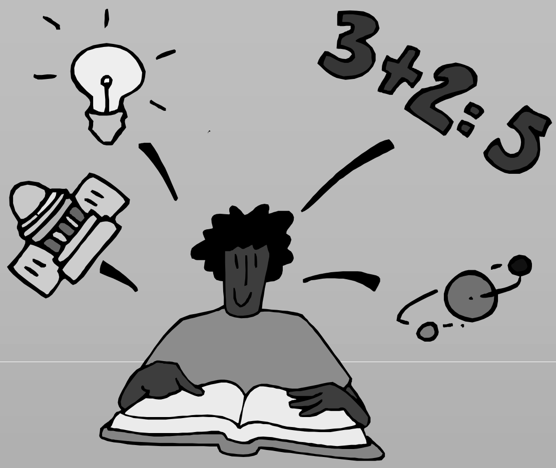
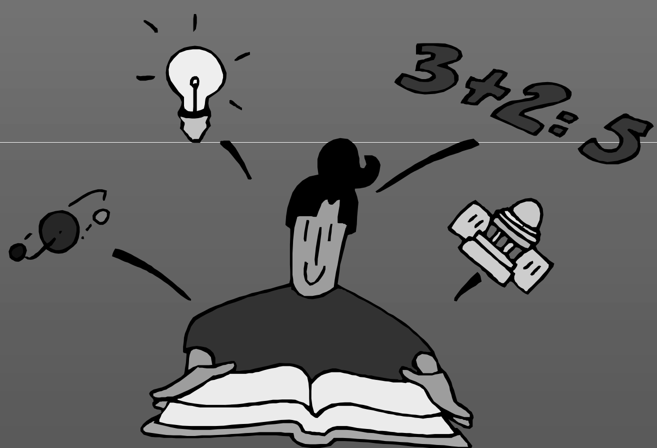
Deptford Green School



The How To Numeracy and Maths Book



CONTENTS

1. NUMERACY METHODS

ESTIMATION

It is very easy to estimate and it is something you should do in both mental and written work. An estimate is a good approximation of a quantity that has been arrived at by judgement rather than guessing. Rounding is used to obtain this good approximation.

Rounding to the nearest ten, hundred or thousand

Remember the rule, 'Five or more'. Look at the next digit after the one to which you are correcting. If this is 5 or more, the digit before goes up.

To the nearest 10	34	becomes	30
	37	becomes	40
To the nearest 100	347	becomes	300
	357	becomes	400
To the nearest whole number	86.4	becomes	86
	86.6	becomes	87

How to use rounding to estimate

e.g.1 $27.6 + (7.2 \times 9.6) \approx 30 + (7 \times 10) = 100$

e.g.2 $\dfrac{57.73 - 25.12}{4.56} \approx \dfrac{60 - 25}{5} = \dfrac{35}{5} = 7$

e.g.3 $\dfrac{18.7}{31.6 - 7.8} \approx \dfrac{20}{30 - 10} = \dfrac{20}{20} = 1$

e.g.4 $105.8 - 5.36^2 \approx 100 - 5^2 = 100 - 25 = 75$

Rounding to 1 significant figure

Usually, the digits in a number, not counting noughts at the beginning are significant figures. Identify the first significant figure. Use the 'Five or more' rule.

681	has 3 s.f.	=	700	1 s.f.
39784	has 5 s.f.	=	40000	1 s.f.
13.06	has 4 s.f.	=	10	1 s.f.

These zeros must be included to keep the answer the correct size

Rounding decimal numbers which lie between 0 and 1 to 1 significant figure

0.900	=	1	1 s.f.
0.0076	=	0.008	1 s.f.

This is the first significant figure

These zeros must be included to keep the answers the correct size

Accuracy including reading a calculator display

To 1 decimal place	45.34	becomes	45.3	1 d.p.
	45.39	becomes	45.4	1 d.p.
To 2 decimal places	45.392	becomes	45.39	2 d.p.
	45.385	becomes	45.39	2 d.p.
	45.395	becomes	45.40	2 d.p.

Workout on your calculator	Write down the calculator display	Write to the nearest whole number	To 1 d.p.	To 2 d.p.
19 ÷ 8	2.375	2	2.4	2.38
269 ÷ 40	6.725	7	6.7	6.73
112 ÷ 23	4.869565217	5	4.9	4.87

Decimals × 10, × 100 and ÷ 10, ÷ 100

Multiplication

Th thousands	H hundreds	T tens	u units	t tenths	h hundredths	th thousandths	
		2	8 •	3	4		×10
	2	8	3 •	4			
		3	1 •	0	6	9	×100
3	1	0	6 •	9			

The decimal point does NOT move. The numbers move to the left in multiplication,

Division

Th thousands	H hundreds	T tens	u units	t tenths	h hundredths	th thousandths	
	8	3	2 •	9			÷10
		8	3 •	2	9		
3	0	6	4 •	8			÷100
		3	0 •	6	4	8	

and to the right in division.

Estimation for decimal multiplication e.g. 8.7 × 6.3

	Method	Example
Step 1	Obtain an estimate for e.g. 8.7 × 6.3	9 × 6 = 54
Step 2	Ignore the decimal point and find the product of the whole numbers either mentally, using Partitioning or Gelosia.	87 × 63 = 5481
Step 3	Compare with the estimate. Insert the decimal point.	**8. 7 × 6. 3 = 54. 81**
Step 4	Compare with your estimate. Is the answer near the estimate? If not, check your working.	54.81 is near 54, so the answer is of the correct order

MULTIPLICATION

Multiplication facts up to 10 x 10 must be learnt or you must be able to derive the answer quickly.

For example if you are asked, 'What is the product of 7 and 8?', you must be able to say,

7 × 8 = 56

and **8 × 7 = 56**

This table should be known by heart.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Doubling and Halving

Most people find Doubling and Halving helps them understand multiplication as well as to be able to calculate the answer quickly. Here is an easy way of remembering the rules.

D = double the number
DD = double the number and double the number again
DDD = double the number, double the answer, and double the answer again
1/2 = multiply by 10 and half the answer
5 +1 = multiply by 10, half the answer and add one lot
5 + d = multiply by 10, half the answer and add a double
10 - 1 = multiply by 10, and subtract one lot

x 2	D
x 3	D + 1
x 4	DD
x 5	1/2
x 6	5 + 1
x 7	5 + D
x 8	DDD
x 9	10 - 1

Here are some examples to help you understand and use these rules.

What is 2 × 7?
Strategy: Double 7

1 × 7 = 14
2 × 7 = 14

∴ **2 × 7 = 14**

What is 4 × 7?
Strategy: Double 7 and double the answer

1 × 7 = 7
2 × 7 = 14
4 × 7 = 28

∴ **4 × 7 = 28**

What is 8 × 7?
Strategy: Double7, double the answer and double again

1 × 7 = 7
2 × 7 = 14
4 × 7 = 28
8 × 7 = 56

∴ **8 × 7 = 56**

What is 6×7?
Strategy: Find $\times$ 10, halve the answer and add 7

$$10 \times 7 = 70$$
$$5 \times 7 = 35$$
$$35 + 7 = 42$$
$$\therefore \underline{\mathbf{6 \times 7 = 42}}$$

What is 7 x 7?
Strategy. Find $\times$ 10, half the answer and add a double

$$10 \times 7 = 70$$
$$5 \times 7 = 35$$
$$7 \times 7 = 35 + 14$$
$$= 49$$
$$\underline{\mathbf{7 \times 7 = 49}}$$

The method can be extended to bigger numbers

e.g. $23 \times 9 = 23 \times (10 - 1) = 23 \times 10 - 23 \times 1 = 230 - 23 = 207$

$$\therefore \underline{\mathbf{23 \times 9 = 207}}$$

e.g. $23 \times 6 = 23 \times (5 + 1) = 23 \times 5 + 23 \times 1 = 115 + 23 = 138$

$$\therefore \underline{\mathbf{23 \times 6 = 138}}$$

The method can be used for even bigger numbers e.g. two, two digit numbers - but many 'stages' have to be held in your head, so multiplying bigger numbers mentally takes lots of practice. The method can be extended to calculations that cannot entirely be done mentally. Just use combinations of facts to work out multiples.

e.g. 46×63

$1 \times 63 =$ 63	or	$1 \times 46 =$ 46
$2 \times 63 =$ 126		$2 \times 46 =$ 92
$4 \times 63 =$ 252		$4 \times 46 =$ 184
$6 \times 63 =$ 276		$3 \times 46 =$ 138
$40 \times 63 =$ 2520		$60 \times 46 =$ 2760
$\therefore$ **$46 \times 63 =$ 2898**		$\therefore$ **$63 \times 46 =$ 2898**

Equivalences

The product of two numbers can sometimes be calculated mentally by spotting equivalences. The rule is that you half one number and double the other.

16×15
8×30
4×60
2×120
1×240 $\therefore$ all these multiplications give a product of 240

and **$16 \times 15 = 240$**

You do not have to just double and half. The method works as long as you increase one number by the same factor as the other is being decreased.

35×18
($\times$ & $\div$ by 2) 70×9
($\times$ & $\div$ by 3) 210×3 $\therefore$ all these multiplications give a product of 630

and **$35 \times 18 = 630$**

Partitioning

Find: 87×63

Step 1 Partition into tens and units and arrange the numbers on a grid.
$87 = 80 + 7$ and $63 = 60 + 3$

Step 2 In each box write the product.

Step 3 Add together the numbers in the boxes.
The total is the product of 87 and 63

	60	+ 3
80		
+ 7		

Write the product of 80 and 3 here

	60	+ 3
80	4800	240
+ 7	420	21

∴ <u>87 × 63 = 548</u>

Gelosia

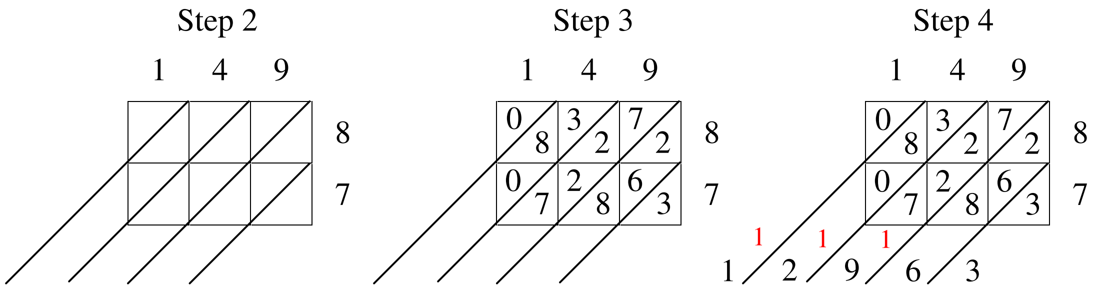
There are many ways to do long multiplication. This method for multiplication is called the lattice method or Gelosia. To multiply 149 by 87, follow these steps;

Step 1 Estimate $149 \times 87 \sim 150 \times 90 = 13\ 500$

Step 2 Draw a grid (see below)

Step 3 Fill in the squares. Each time multiply the number at the top by the one at the side.
For example $9 \times 8 = 72$ and $4 \times 7 = 28$

Step 4 Add diagonally



∴ <u>123 × 87 = 12 963</u>

ADDITION

Adding-on (Continental two-step)

The best way to add and subtract is by written informal methods. Rarely is a formal (vertical) method needed. Very large numbers will most likely occur in problems best suited for a calculator. You must be able to derive quickly

Doubles (of all numbers to 100)
Near doubles
Bonds to 100
Addition of single digit numbers
Addition of two, two digit numbers and use these strategies in problems.

Estimate the answer. Then, start with the bigger number because it is nearer the answer.

Find	Working out	Answer
43 + 38	43 + 38 = 73 + 8	**43 + 38 = 81**
75 + 189	189 + 75 = 259 + 5	**75 + 189 = 264**
498 + 549	549 + 498 = 949 + 98 949 +100 -2 = 1047	**498 + 549 = 1047**
355 + 478	478 + 355 = 778 + 55	**355 + 478 = 833**

Equivalences

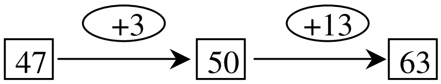
Increase one number, decrease the other by the same amount

Find	Working out	Answer
43 + 38	43 + 38 41 + 40 = 81	**43 + 38 = 81**
75 + 189	75 + 189 64 + 200 = 264	**75 + 189 = 264**
498 + 549	498 + 549 500 + 547 = 1047	**498 + 549 = 1047**
355 + 478	355 + 478 350 + 483 = 783 + 50 = 833	**355 + 478 = 833**

SUBTRACTION

<u>Next Ten</u>

Find 63 - 47

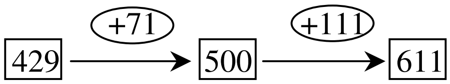


∴ **<u>63 - 47 = 16</u>**

Method

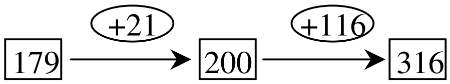
Round 47 to the next 10.
Put next ten in box and the number added in the circle.
Work out how many needs to be added to get the answer. Put in ring.
Add the numbers in the circle mentally. (If not set up another diagram.)

Find 611 - 429



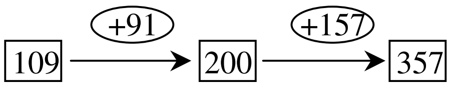
∴ **<u>611 - 429 = 182</u>**

Find 316 - 179



∴ **<u>316 - 179 = 137</u>**

Find 357 - 109



∴ **<u>357 - 109 = 248</u>**

Alternatively, still using the method of rounding to the next ten, the working out can be set out like this:

272 - 28 = 272 - 30 + 2 = 242 + 2 = 244 ∴ **<u>272 - 28 = 244</u>**
275 - 42 = 275 - 50 + 8 = 225 + 8 = 223 ∴ **<u>275 - 42 = 223</u>**
275 - 133 = 275 -140 + 7 = 135 + 2 = 137 ∴ **<u>275 - 133 = 137</u>**

<u>Equivalences</u>

Both numbers increase or decrease together

Find	Working out	Answer
63 - 47	63 - 47 66 - 50 = 16	∴ **<u>63 - 47 = 16</u>**
611 - 429	611 - 429 612 - 430 = 182	∴ **<u>611 - 429 = 182</u>**
316 - 179	316 - 179 337 - 200 = 137	∴ **<u>316 - 179 = 137</u>**
357 - 109	357 - 109 348 - 100 = 248	∴ **<u>357 - 109 = 248</u>**

DIVISION

Inverse of multiplication

Division can be accomplished by knowing your multiplication tables and understanding that division is the inverse process of multiplication.

$$\underset{\text{Dividend}}{\underset{\uparrow}{56}} \quad \div \quad \underset{\text{Divisor}}{\underset{\uparrow}{8}} \quad = \quad \underset{\text{Quotient}}{\underset{\uparrow}{7}} \qquad \text{because } 7 \times 8 = 56$$

Double and add to get close

Division can also be accomplished by reversing the process of doubling and halving.

Find $96 \div 8$

	$1 \times 8 = 8$	Step 1	Look for products that add up to 96
	$2 \times 8 = 16$	Step 2	Tick off line containing these products
✓	$4 \times 8 = 32$	Step 3	Add up the multipliers
✓	$8 \times 8 = 64$		

$96 \div 8 = 12$

If the products do not add up exactly to the dividend, find the product sum just smaller than the dividend, and express the remainder as a fraction of the divisor.

Find $114 \div 7$

	$1 \times 7 = 8$
✓	$2 \times 7 = 16$
✓	$4 \times 7 = 32$
✓	$10 \times 7 = 70$

$(114 - 112 = 2)$

$\therefore$ **$114 \div 7 = 16\frac{2}{7}$**

Find $567 \div 24$

✓	$1 \times 24 = 24$
✓	$2 \times 24 = 48$
✓	$20 \times 24 = 480$

$(567 - 552 = 15)$

$567 \div 24 = 23\frac{15}{24}$

$\therefore$ **$567 \div 24 = 23\frac{5}{8}$**

Find $612 \div 27$

	$1 \times 27 = 27$
✓	$2 \times 27 = 54$
✓	$20 \times 27 = 540$

$(612 - 594 = 18)$

$612 \div 27 = 22\frac{18}{27}$

$\therefore$ **$612 \div 27 = 22\frac{2}{3}$**

This method is efficient:

- To use mentally
- As a written method
- With or without a remainder

Divisibility Tests

Knowing the divisibility rules gives you another tool in your quest to find accurate answers to division. These rules inform you if a number can be divided exactly without a remainder.

E: The number is even EE: The number is even and half the number is even EEE: The number is even, half the number is even and half of the half is even Digit sum: Add together all the digits of a number, then add the digits of the answer, and so on, until you end up with a single digit. This number is called the digit sum. e.g. the digit sum of 365 is 5 3 + 6 + 5 = 14 1 + 4 = 5 0: The number ends in 0 5: The number ends in 5	÷ 2 ÷ 3 ÷ 4 ÷ 5 ÷ 6 ÷ 7 ÷ 8 ÷ 9 ÷ 10	E Digit sum ÷ 3 EE 0,5 E + Digit sum ÷3 No test EEE Digit sum ÷ 9 0

Use the rules to check if these numbers have a remainder.

e.g. 1. Is there a remainder when 216 is divided by 8?
216 is even
half the number, 108, is even
and half of that, 54, is even — No remainder
216 is divisible by 8 (and 2 and 4)

e.g. 2. Is there a remainder when 729 is divided by 9?
The digit sum is 9 (1 + 8)
there is no remainder when divided by 9 — No remainder
729 is divisible by 9 (and 3)

e.g. 3. Is there a remainder when 324 is divided by 6?
324 is even ∴ it is divisible by 2.
The digit sum of 324 is 9, ∴ the number is divisible by 3
Therefore 324 is exactly divisible by 6. — No remainder
324 is divisible by 6 (and 2 and 3)

Division by a decimal

Never divide by a decimal. First turn the divisor into a whole number by multiplying by 10,100, 1000 etc

e.g. What is $1.235 \div 0.05$?

Multiply the divisor by 100 $\quad 0.05 \times 100 = 5$

Multiply the dividend by same number, the problem becomes $123.5 \div 5$

Here are some other examples

3.69 ÷ **0.3**	× by 10	36.9 ÷ **3**	=	12.3
43.55 ÷ **1.5**	× by 10	435.5 ÷ **15**	=	29.03
7.2 ÷ **0.09**	× by 100	720.0 ÷ **9**	=	80
9.464 ÷ **0.26**	× by 100	946.4 ÷ **26**	=	36.4
62.5 ÷ **0.005**	× by 1000	62500 ÷ **5**	=	12500

Using factors

What is 342 divide by 18?
Split 18 into its prime factors $18 = 2 \times 3 \times 3$

(÷ 2) $342 \div 2 = 171$
(÷ 3) $171 \div 3 = 57$
(÷ 3) $57 \div 3 = 19$

342 ÷ 18 = 19

- This method is useful when there is no remainder (use divisibility rules to check)
- The divisor can be factorised (i.e. not prime)

Equivalences

Spotting equivalences often helps with division. The rule is that you either divide both numbers by the same factor or multiply both numbers by the same factor. The answer stays the same

900 ÷ 36
(÷2) 450 ÷ 18
(÷3) 150 ÷ 6
(÷3) 50 ÷ 2
(÷2) 25 ÷ 1

All of these divisions give a quotient of 25

∴ **900 ÷ 36 = 25**

114 ÷ 6
(÷3) 38 ÷ 2
19 ÷ 1

All of these divisions give a quotient of 19

∴ **114 ÷ 6 = 19**

Equivalences are particularly efficient when multiplying by 25.
To multiply by 25, multiply by 100 and divide by 4

25 × 44 = 4400 ÷ 4
= 2200 ÷ 2
= 1100 ÷ 1

25 × 44 = 1100

25 × 237 = 23700 ÷ 4
= 11850 ÷ 2
= 5925 ÷ 1

25 × 237 = 5925

Division Algorithm

Dividing by a single digit when there is no remainder

$3\overline{)69}$ → $3\overline{)6\;9}$ = 2 3 → $\therefore$ **69 ÷ 3 = 23**

$3\overline{)45}$ → $3\overline{)4\,{}^{1}5}$ = 1 5 → $\therefore$ **45 ÷ 3 = 15**

Do not put remainders at the end of a decimal division. At the end of every whole number there is an imaginary decimal point. So, put in the decimal point and add as many noughts as you need.

Do not write this: ~~$4\overline{)846}$ = 211r2~~

$4\overline{)846.{}^{2}0}$ = 211. 5 → $\therefore$ **846 ÷ 4 = 211.5**

Line up decimal points

Put in as many noughts as you need

Find 7.14 ÷ 3

$3\overline{)7\,.\,1\,4}$ (point placed above) → $3\overline{)7\,.\,{}^{1}1\,{}^{2}4}$ = 2 . 3 8

Keep the point in the answer above the point in the question

$\therefore$ **7.14 ÷ 3 = 2.38**

24.3 ÷ 5

$5\overline{)2\,4\,.\,3}$ → $5\overline{)2\,4\,.\,{}^{4}3\,{}^{3}0}$ = 4 . 8 6

$\therefore$ **24.3 ÷ 5 = 4.86**

Add noughts …as many as you need

2.9 ÷ 8

$8\overline{)2\,.\,9}$ → $8\overline{)2\,.\,{}^{2}9\,{}^{5}0\,{}^{2}0\,{}^{4}0}$ = 0 . 3 6 2 5

$\therefore$ **2.9 ÷ 8 = 0.3625**

The 6 is a recurring number

0.47 ÷ 3

$3\overline{)0\,.\,4\,7}$ → $3\overline{)0\,.\,4\,{}^{1}7\,{}^{2}0\,{}^{2}0\,{}^{2}0}$ = 0 . 1 5 6 6 6

0.47 ÷ 3 = 0.15666…

$\therefore$ **0.47 ÷ 3 = $0.15\dot{6}$**

The 4 and 5 are recurring numbers

6.43 ÷ 11

$11\overline{)6\,.\,4\,3}$ → $11\overline{)6\,.\,4\,{}^{1}3\,{}^{2}0\,{}^{2}0\,{}^{2}0\,{}^{2}0\,{}^{2}0\,{}^{2}0}$ = 0 . 5 8 4 5 4 5 4 5

6.43 ÷ 11 = 0.584545..

$\therefore$ **6.43 ÷ 11 = $0.58\dot{4}\dot{5}$**

2: ALGEBRA

WRITING ALGEBRA

Writing algebra is like writing a computer programming language. Even a tiny mistake can make the whole thing meaningless. You must write algebra neatly and accurately.

- Upper case and lower case letters are different:
  In general little 'a' is not the same as big 'A'
- Some letters can easily look like numbers i and j look like 1, so make the dot very clear and z looks like 2, so it's best to cross your ~~z~~'s.
- Use a curly x so it isn't confused with a multiplication sign ×
- Powers like squared x^2 and cubed x^3 are always about half sized and raised.
- Don't confuse with sequences where the second term is u_2 the '2' is small and lowered.
- Always write fractions with a horizontal line $\frac{2}{3}$, then they are the same as algebraic fractions $\frac{x+2}{3x}$.

<u>The '=' Equals Sign</u>

Be careful with chains of equals signs. Make sure that everything stays equal!

This is wrong 25 – 17 = 25 – 10 = 15 – 7 = 8

- In the middle it says 25 – 10 = 15 – 7 which is NOT true!
- Use arrows to show your working if it helps:

  25 – 17 → 25 – 10 → 15 – 7 = 8

Try to avoid chains altogether. This would be best:

$$\begin{aligned} 25 - 17 &= 25 - 10 - 7 \\ &= 15 - 7 \\ &= \underline{8} \end{aligned}$$

Doing Algebra

- Work step-by-step.
- Make a note to say what you did in each step.
- Work down the page.
- Keep equals signs in a vertical line.
- Underline the answer.

Solve $3x + 4 = 22$

$$3x + 4 = 22$$
$$3x = 18 \quad (-\ 4)$$
$$\underline{x = 6} \quad (\div 3)$$

Notes to say what happened in each step

Underlined answer

Equals signs in a vertical line

Partitioning in Algebra

Multiplying out brackets

$(2x+3)(x-5)$

	$2x$	$+3$
x	$2x^2$	$3x$
-5	$-5x$	-15

Multiply to fill in the boxes

Add up the two '*x*' parts

$2x^2 + 3x - 5x - 15 = \underline{2x^2 - 2x - 15}$

Factorising

$x^2 + 10x + 16$

You should spot: $8 + 2 = 10$ and $8 \times 2 = 16$

	x	$+8$
x	x^2	$8x$
$+2$	$2x$	16

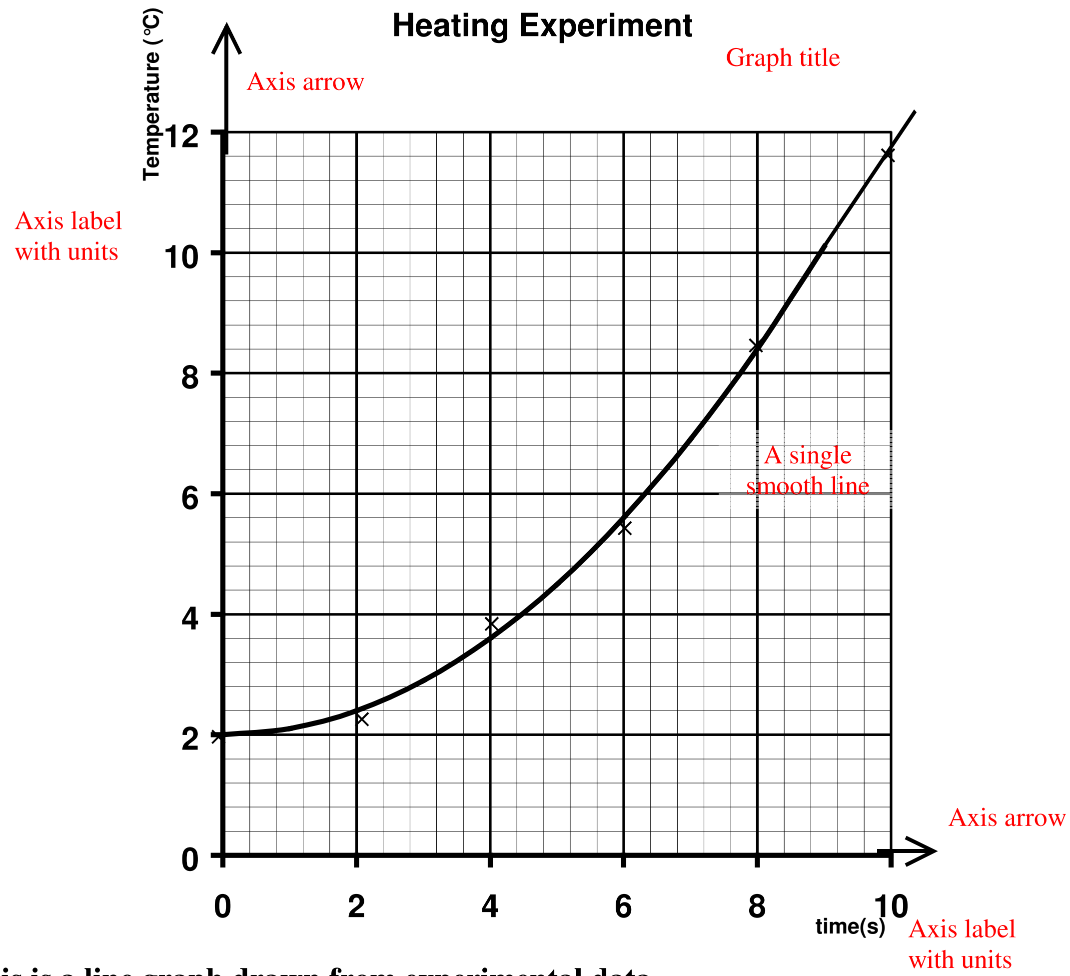
Fill in the x^2 and the 16. Put $2x$ and $8x$ because you spotted that it must be 8

Now you can fill in the outsides of the table

$x^2 + 10x + 16 = \underline{(x+8)(x+2)}$

3. CHARTS AND GRAPHS

GRAPHS



This is a line graph drawn from experimental data

On computer software such as Excel it is called an *x*/*y* plot.

- Plot each of your points with a neat cross
- Make a **single smooth line** which comes as close as possible to passing through all of the points.
- **DO NOT** join the points in dot-to-dot fashion!
- Extend the line beyond the last point.

All graphs must have:

- A title
- Labels on both of the axes
- Arrows at the end of both axes

STATISTICAL CHARTS

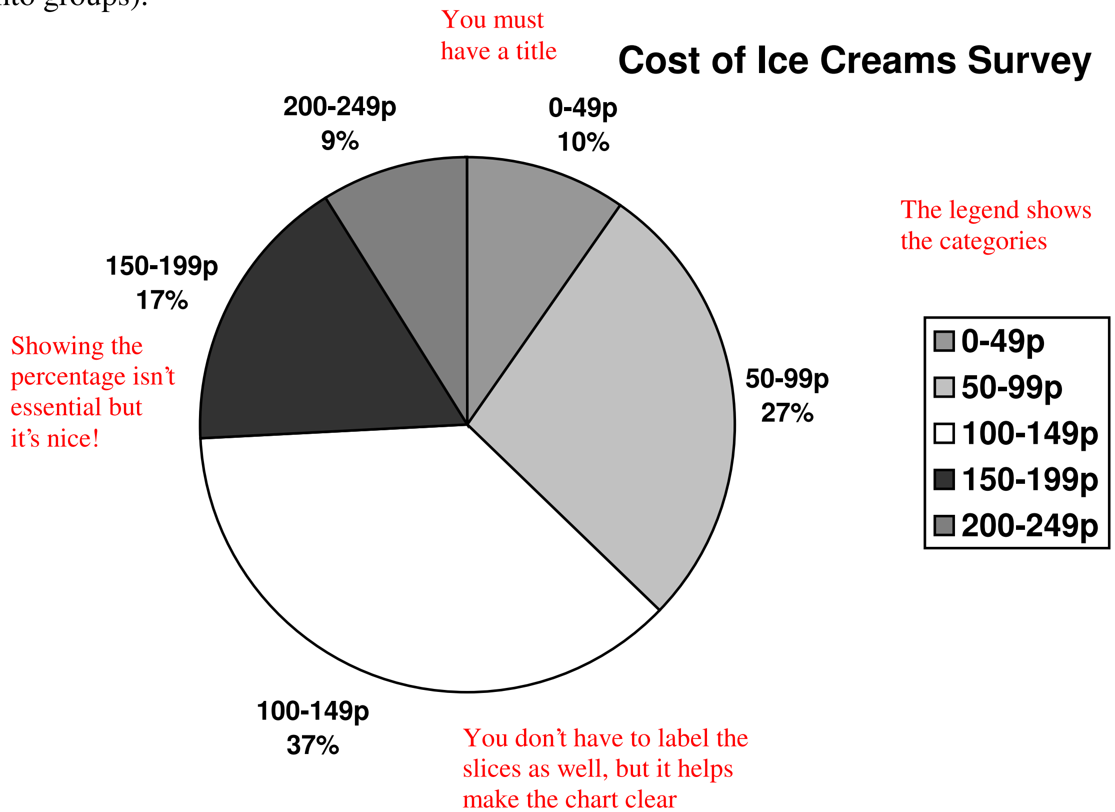
It is important to choose the correct chart for your data.

We have to decide what type of data we have:
Categorical: a list of categories – (t doesn't matter what order they come in)
e.g.1: favourite chocolate bars – twix, mars, kit-kat, lion bar.
e.g.2: vehicles – car, van, lorry, bus, bicycle.

Discrete: a list of numbers where every amount is known exactly.
e.g. 1: the prices of stamps – 2p, 3p, 5p, 12p, 18p, 25p, 30p
e.g. 2: shoe sizes – 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, …..

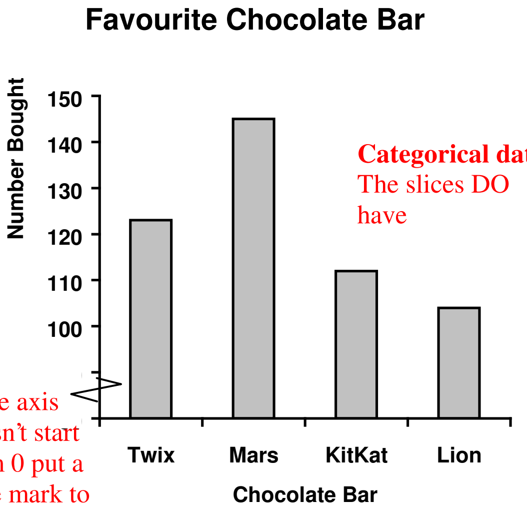
Continuous: numbers which could have any value, most commonly anything you have to measure, like time or distance.
e.g. 1: heights of people – 1.45m (we have to write it to 2 decimal places)
e.g. 2: 100m sprint times – 9.897s (we have written this to 3 d.p.)

Pie Chart
You should only use pie charts for categorical data.
(You can turn discrete or continuous data into categorical data by putting the numbers into groups).



Bar Chart

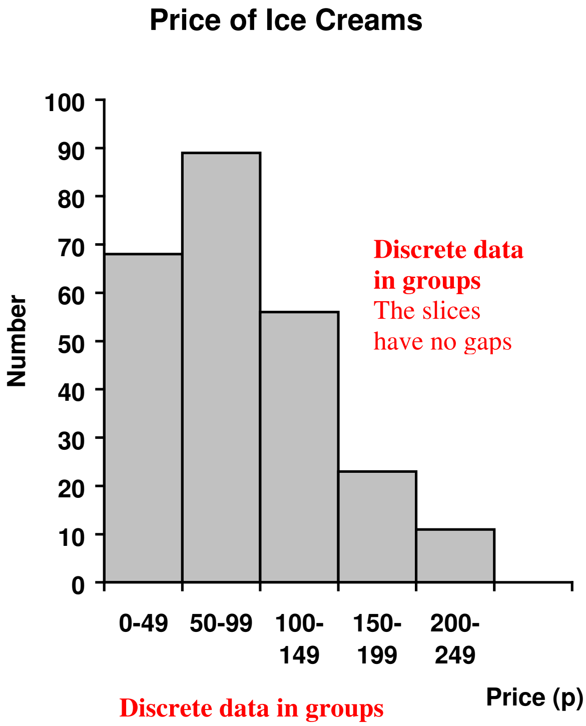
Also called: bar graph, column graph, block graph.



Categorical data
The slices DO have

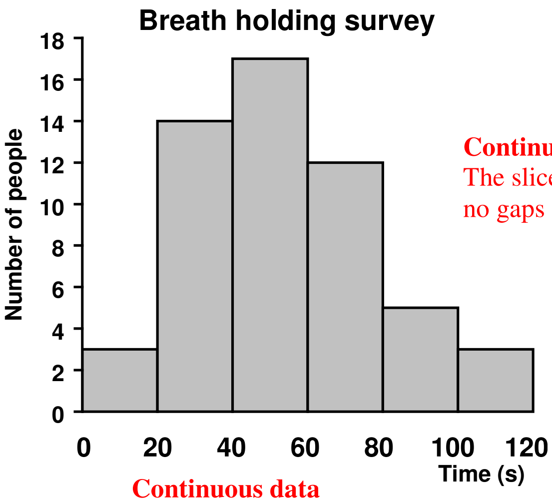
If the axis doesn't start from 0 put a little mark to show a 'broken

Categorical data
The axis labels go in the middle of the bars



Discrete data in groups
The slices have no gaps

Discrete data in groups
The axis labels go in the middle of the bars



Continuous data
The slices have no gaps

Continuous data
The axis labels begin and end with the bars – like a graph

All charts must have:

- A title
- Labels on both of the axes

NOTES